Math 152 Project 2   Due: Wednesday 4/9

This project will focus on applications of derivatives (aka, rates of change). Specifically, we will use the ideas of Chapter 20 regarding maximums and concavity.

Background: A long time ago in a galaxy far away there existed a group of 9 individuals who loved nothing more than to study mathematics. Their country of origin had a modest population of 150 people, which made this group the leading mathematicians in their homeland. Despite their best efforts to educate them, their countrymen insisted on dividing by zero and most had even experimented with *taking the square root of negative numbers*. Mathematically speaking, the country was a hot mess. The members of the group were so fed up with their situation that they packed a boat with all the necessary materials and set off in search of a new homeland. Lucky for them, their country was surrounded by countless bountiful islands. After six days of travel without seeing any land at all, the group found a beautiful and uninhabited island. The group spent the first few days setting up their colony and surveying the surrounding land. Although the island was very small, the group realized that there was enough room and sustainable resources for their population to survive and grow. Excited by the prospect of creating a utopia of mathematicians, Exponential G. Rowth, the group leader, exclaimed "We have found a home far from civilization and bountiful with resources. Assuming we are left alone, our population will one day rival that of our homeland!" A rising star of the group, Optimus von Calculus, was not so sure about this claim. Help Optimus explore this question by considering the following model:

The population of the group is modeled by

$$P(t) = 70e^{-2e^{-0.05t}}$$

Where P is the population and t is measured in years since the group found the island.

Key Assumptions of the Model

- The island is uninhabited which means there is room for the initial population to grow

- Renewable resources (animals, plants, water etc) allow the initial population to grow

- The island has a limited amount of space

- The island has a limited amount of resources

- The growth of the population is constrained by the limited amount of space and resources

- The growth in the model assumes no extraneous outside factors impact the population growth. For example: disease, weather, wars or anything else that would impact the increase/decline of a population

Things to Investigate:

1) What is the largest size that the population can achieve?

2) When does the growth of the population stop increasing?

3) What does the population curve look like? (use this to verify results from 1) and 2))

Investigation 1: What is the largest size the population can achieve?
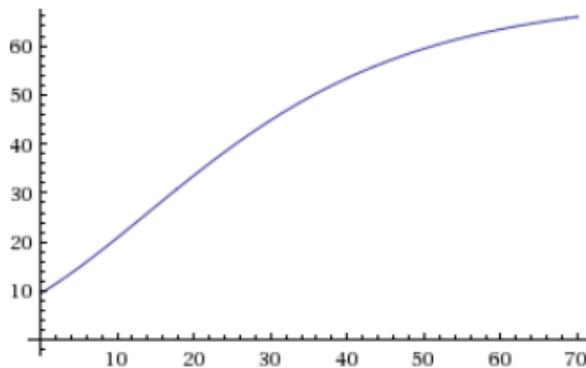
The function $P(t)$ does not have a true maximum (ie, $P'(t) \neq 0$ for any t). Instead, it approaches a maximum asymptotically. For this reason, to find the max population we should see what happens to P as $t \to \infty$.

That is, determine $\lim_{t \to \infty} 70e^{-2e^{-0.05t}}$

Graded Item #1: Documentation of the above limit using a MATLAB script.

Graded Item #2: Answer the following question:

Remember that the group's homeland had a population of 150. Will the islands population actually ever exceed 150? Explain.

(There is information about the steps you need to take to complete this project on page 5&6)

Investigation 2: When does the growth of the population stop increasing?
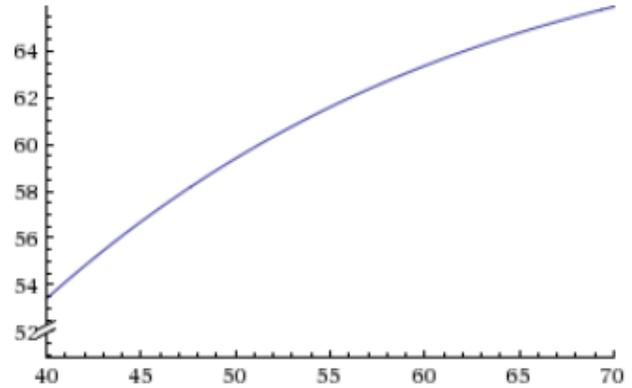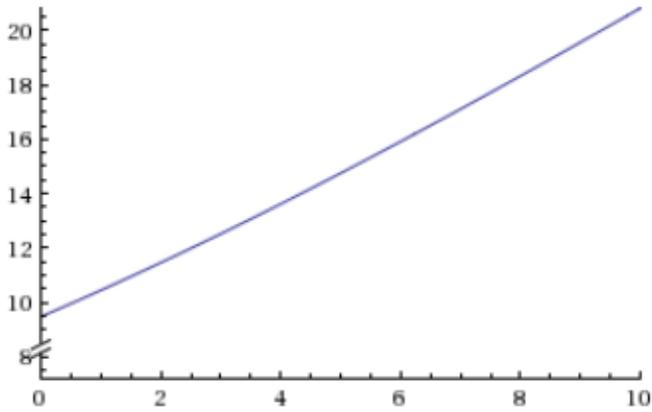
Although the population is constantly increasing as time goes on, by examining the above graphs it is clear that the rate at which the population grows increases initially, but eventually begins to decrease.

The switch from $P'(t)$ increasing to $P'(t)$ decreasing takes place when $P''(t) = 0$.

To determine when this happens you need create a MATLAB script that defines the function $P(t)$, takes two derivatives of $P(t)$ and then solves $P''(t) = 0$.

Graded Item #3: Determine the time at which the growth of the population starts to decrease. Include a printout of your MATLAB script as well as the output the script produces. What was the size of the population at this time?

Graded Item #4: We have seen here that the rate of change of the population changes from increasing to decreasing when the second derivative is zero. That is, $P''(t) = 0$ corresponds to $P'(t)$ changing from increasing to decreasing. Explain why this is true.

Hint: $P''(t)$ tells us the same things about $P'(t)$ as $P'(t)$ tells us about $P(t)$.

(There is information about the steps you need to take to complete this project on page 5&6)

Investigation 3: What does the population curve look like?

Plot $P(t)$ from $t = 0$ to $t = 150$ to visually verify your results from the previous 2 investigations. Circle the inflection point found in investigation 2.

Graded Item #5: Print out of the above graph. Have an appropriate title, label each axis and circle the point of inflection.

Graded Item #6: We assumed that the population survived and grew in response to available space remaining on, and resources produced by, the island. Why was Exponential G. Rowth wrong in his claim that the group would eventually have a population greater than 150? What did he fail to consider?

Graded Item #7: In order to make modelling the population possible, we over simplified a real-world environment by making many assumptions. Edit and/or add at least two assumptions and explain how they would better reflect a real-world environment.

Grading Rubric for Math 152 Project 2- Each of the following item is worth 5 points. 40 points total.

Graded Item	Explanation	Points Earned	Graded Item	Explanation	Points Earned
#1	matlab printout illustrating the limit		#5	Graph with title, axis and inflection point	
#2	Using the answer to #1 to appropriately explain the answer		#6	Logical thoughts, complete sentences etc	
#3	matlab script and output of the script		#7	Logical thoughts, complete sentences etc	
#4	Use ideas from ch 20 on increasing and decreasing to relate to P'' and P'		**Report Organization and neatness**	The report should be on clean sheets of paper, be neat and in an appropriate order	

Bonus Question (+3 points): If you try to solve $P'(t) = 0$ in MATLAB it gives the error

Warning: Explicit solution could not be found.

Explain why.

What you need in order to do this project

A) Entering the function itself into a MATLAB script can be tricky. Here is how it should be entered:

```
P=@(t) 70*exp(-2*exp(-.05*t));
```

B) How to evaluate the limit $\lim_{t \to \infty} 70e^{-2e^{-0.05t}}$.

We worked on how to do this at the beginning of the semester. Project 1 also involved finding an infinite limit in MATLAB. None of our algebraic techniques will work for this function so you should adjust your MATLAB code from project 1 to work for the function in this project.

C) Finding derivatives in MATLAB

   and

D) Solving an equation in MATLAB

As you can see in the graph at right, $f(x) = x^3 - 6x^2$ has an inflection point at $x = 2$. That is, $f''(2) = 0$
The following matlab code shows how to find this
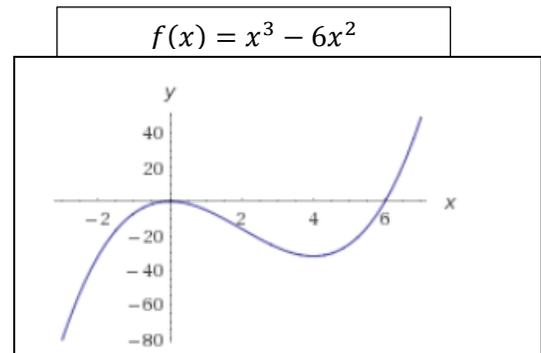
$$f(x) = x^3 - 6x^2$$


```
         FILE                          EDIT           NAVIGATE    BREAKPOINTS                        RUN
  Sp14Project2Example.m   X   +
1
2    syms x       %tells matlab that it should consider x a variable
3                 % this needs to be at the top of the script
4
5    f = @(x) x.^3-6*x.^2;   %this defines the function
6
7
8    FirstDeriv=diff(f,x)     %diff(f,t) takes the derivative of the function f
9                             %respect to its variable, x.
10                            %by writing FirstDeriv=diff(f,x) we are storing the
11                            %derivative as "FirstDeriv"
12
13   SecondDeriv=diff(FirstDeriv,x)     %this takes the second derivative and
14                            %stores it as SecondDeriv
15
16   InflectionPoint=solve(SecondDeriv==0) %solves f''=0  and stores it in
17                            % "InflectionPoint"
18                            %note the double equal sign
19
20   %any line that does not contain a ";" at the end will print to the command
21   %window
```

What prints to the command window when you run the previous page's script:
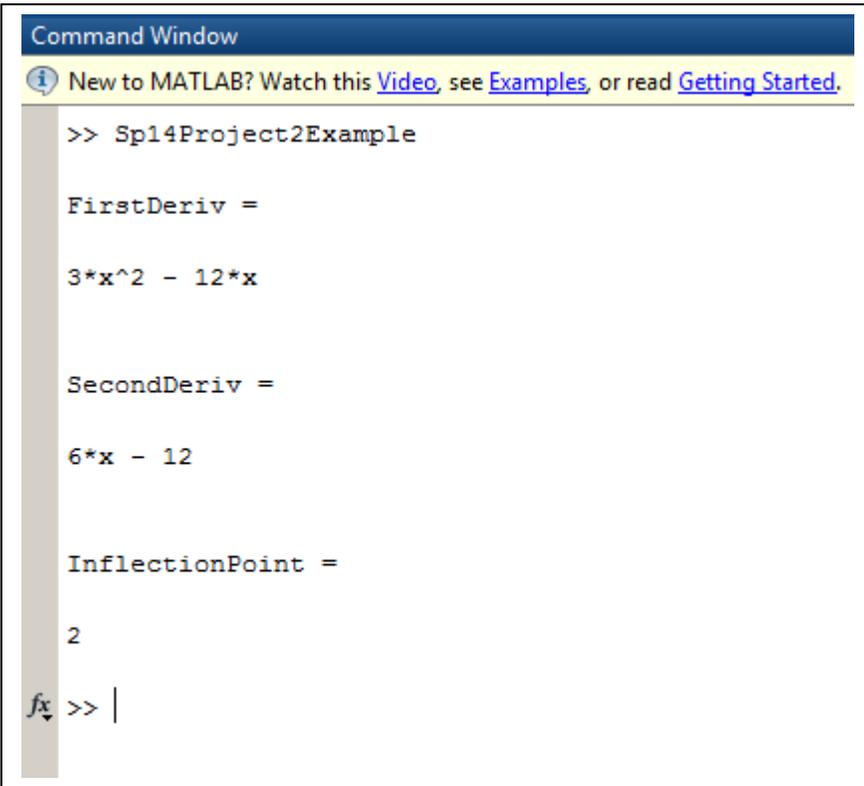
```
Command Window
ⓘ New to MATLAB? Watch this Video, see Examples, or read Getting Started.
  >> Sp14Project2Example

  FirstDeriv =

  3*x^2 - 12*x


  SecondDeriv =

  6*x - 12


  InflectionPoint =

  2

fx >> |
```

E) Plotting $P(t)$

You have learned how to plot previously, including in project 1. Here is an example code that plots the previous function:

```
24   f = @(x) x.^3-6*x.^2;   %this defines the function
25
26   x=-2:.1:8;       %gets x values ready to plug into the function
27
28   y=f(x);          % plugs all x values from line 26 into the function and
29                    % stores them in "y"
30
31   plot(x,y);       %plots x and y which are both vectors from lines 26 and 28.
```